Norddeutsches Gruppentheorie-Kolloquium

8-9 November 2024

Schedule

Friday 8 November	
12:00 – 13:00	Registration and refreshments
13:00 – 13:40	Attila Maroti: *On the Gowers trick for classical simple groups*
13:55 – 14:35	Sebastian Bischof: *Roots of flat groups and Bruhat-Tits buildings*
Tea/Coffee/Refreshments	
15:10 – 15:50	Ralf Köhl: *Lattices of minimal covolume in simple Lie groups*
16:05 – 16:45	Ettore Marmo: *Toric arrangements and Bloch-Kato pro-p groups*
Tea/Coffee/Refreshments	
17:00 – 17:40	Bettina Eick: *Finite p-groups of maximal class with 'large' automorphism group*
19:00	Dinner (Grubers Welt)

Saturday 9 November	
09:00 – 09:40	Eamonn O'Brien: *Conjugacy in finite groups*
Tea/Coffee/Refreshments	
10:10 – 10:50	Caroline Lassueur: *On the Trivial Source Character Tables Of Finite Groups*
11:05 – 11:45	Martin van Beek: *Fusion Systems and Simple Groups With Class Two Sylow p-subgroups*
Tea/Coffee/Refreshments	
12:20 – 13:00	Chris Parker: *Fusion Systems on polynomial p-groups*

Talks

Sebastian Bischof

Title: *Roots of flat groups and Bruhat-Tits buildings*

Abstract: For a long time, van Dantzig's theorem (the set of compact open subgroups forms a neighborhood basis of the identity) was the only deep result for general totally disconnected locally compact (tdlc) groups. In his groundbreaking paper from 1994, Willis introduced the *scale* of an automorphism α of a tdlc group G as

$$s(\alpha) := \min\{[\alpha(U) : \alpha(U) \cap U] \mid U \leq G \text{ compact and open}\}.$$

We call a compact open subgroup U *minimizing for* α if the minimum is attained at U. A subgroup $\mathscr{H} \leq \mathrm{Aut}(G)$ is called *flat*, if there exists a subgroup which is minimizing for all $\alpha \in \mathscr{H}$. Willis has shown that to any flat group $\mathscr{H}$ one can associate a set $\Phi(\mathscr{H})$ of *roots* where a root is a suitable homomorphism $\ell : \mathscr{H} \to \mathbb{Z}$.

In this talk we will discuss a strategy to compute $\Phi(\mathscr{H})$ for a given flat group $\mathscr{H}$. Moreover, we will consider the case of Bruhat-Tits buildings in more detail and compare the two notions of roots

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Martin van Beek

Title: *Fusion Systems and Simple Groups With Class Two Sylow p-subgroups*

Abstract: We determine all reduced saturated fusion systems supported on a finite p-group of nilpotency class two. As a consequence, we obtain a new proof of Gilman & Gorenstein's classification of finite simple groups with class two Sylow 2-subgroups.

Bettina Eick

Title: *Finite p-groups of maximal class with 'large' automorphism group*

Abstract: A finite group G of prime-power order p^n has maximal class if its nilpotency class is $n-1$. The p-groups of maximal class have been investigated extensively in the literature. For primes $p = 2$ and $p = 3$ they are fully classified, for prime $p = 5$ there is a detailed conjectural classification available, for all primes $p \geq 7$ the classification of p-groups of maximal class is wide open.

A finite p-group G of maximal class has a 'large' automorphism group if it satisfies $(p-1) \mid |\mathrm{Aut}(G)|$. We consider the graph $\mathcal{G}_p$ associated with such groups: the vertices of this graph correspond one-to-one to the isomorphism types of considered groups and there is an edge $G \to H$ if $H/\gamma(H) \cong G$, where $\gamma(H)$ is the last non-trivial term of the lower central series.

In this talk we exhibit the structure of $\mathcal{G}_p$. This structure translates to a broad classification of the p-groups of maximal class with 'large' automorphism group.

(Joint work with Heiko Dietrich)

Ralf Köhl

Title: *Lattices of minimal covolume in simple Lie groups*

Abstract: The additive group $(\mathbb{R}^n, +)$ admits $((\varepsilon\mathbb{Z})^n, +)$ as lattices, which can realize arbitrarily small covolume.

In simple Lie groups on the other hand such examples cannot exist by theorems by Kazhdan-Margulis and Wang; on the contrary, there in fact exist lattices of minimal covolume. In my talk I will discuss the (cocompact) lattice realizing minimal covolume in $\mathrm{SL}(2, \mathbb{R})$ constructed by Siegel related to the hyperbolic triangle of type $(2, 3, 7)$ and I will describe the current state of research; in particular, Thilmany proved that for $n > 2$ the (non-cocompact) lattice $\mathrm{SL}(n, \mathbb{Z})$ realizes minimal covolume in $\mathrm{SL}(n, \mathbb{R})$ and Džambić, Holm, K proved that for $n \geq 2$ the (non-cocompact) lattice $\mathrm{Sp}(2n, \mathbb{Z})$ realizes minimal covolume in $\mathrm{Sp}(2n, \mathbb{R})$.

Caroline Lassueur

Title: *On the Trivial Source Character Tables Of Finite Groups*

Abstract: The aim of this talk is to review results obtained towards the calculation of trivial source character tables of "small" finite groups, and the creation of a database of such tables.

Ettore Marmo

Title: *Toric arrangements and Bloch-Kato pro-p groups*

Abstract: For every field K we can define a distinguished extension K^{sep} called its separable closure. The maximal pro-p quotient $G_K(p)$ of the Galois group $G_K = \mathrm{Gal}(K^{\mathrm{sep}}/K)$ is called the maximal pro-p Galois group of K, many arithmetical properties of the field are encoded in the structure of this group.

It is interesting to ask which pro-p groups can be realized as the maximal pro-p Galois group of some field. It is known that any such group must also satisfy a cohomological property called the Bloch-Kato property.

In this talk we will discuss some families of pro-p groups arising from hyperplane and toric arrangements and some techniques to study the Bloch-Kato property in this context. This talk is based on joint work with Th. Weigel and E. Delucchi.

Attila Maroti

Title: *On the Gowers trick for classical simple groups*

Abstract: Let G be a finite group and let k be the minimal degree of a nontrivial complex (irreducible) character of G. The Gowers trick states that if A, B, C are subsets of G such that $|A||B||C| > |G|^3/k$, then $ABC = G$. In this talk we will present a variant of this theorem in the special case when G is a classical simple group, the sets A, B, C are relatively large and at least two of them are normal (subsets) in G. This is joint work with Francesco Fumagalli.

Eamonn O'Brien

Title: *Conjugacy in finite groups*

Abstract: We report on the successful outcome of a project to provide complete theoretical and practical solutions to conjugacy problems in finite classical groups.

In particular we can list all classes; construct centralisers; and decide constructively conjugacy between elements of all classical groups. We discuss the implications of this work for the identical problems in finite groups. This is joint work with Giovanni de Franchesci and Martin Liebeck.

Chris Parker

Title: *Fusion Systems on polynomial p-groups*

Abstract: In this talk, I will define polynomial p-groups, review the known fusion systems supported by these groups, and present recent results by Grazian, Parker, Semeraro, and van Beek on the classification of all saturated fusion systems supported by polynomial p-groups.
